The Application of the High-Speed Pixel Clustering Method in Combining Multi-Angle Images Obtained from Airborne Optical-Location Systems

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***Abstract*—This paper deals with the problem of complexing images formed by the onboard equipment of a multi-position optical-location system. The problem of information aggregation is solved quite difficult in airborne optical-location systems. Therefore, in order to combine data received from high-resolution optical sensors into a single information field, the method based on the developed high-speed method of clustering image pixels is proposed. The purpose of present paper is to develop a method for combining multi-angle images, which is characterized by gradual detailing of clustered images.**

The result of present paper is a data aggregation technique that allows combining high-resolution multi-angle location images into a single information field with high correlation characteristics as compared to classical algorithms of combining. The practical relevance of paper involves the feasibility of applying the technique of combining digital images for modern optical monitoring systems designed for environmental exploration, forecasting and rapid prevention of natural and man-made emergencies.

***Keywords* — image segmentation, pixel clustering, multi-angle images, image alignment, optical-location system, full-size images, reference points, alignment by contour points, information aggregation, small aircraft, multi-position system.**

I. Introduction

At the state-of-the-art of development of science and technology, multi-position location small-sized mobile systems are increasingly used for environmental exploration of hard-to-reach places (zones that pose an environmental hazard), as well as for rapid prevention of natural and man-made emergencies. These systems are usually based on small unmanned vehicles (UVs). This is due to a number of reasons. First, the tactical and technical characteristics of the UV allow to use an increasing payload. Secondly, on-board location systems are reduced in their weight and size characteristics, which leads to an increase in the flight time of the UV. These reasons allow UV to carry out rapid and long-term monitoring. However, for operational purposes of warning about emergencies and other environmental disasters, it is advisable to use several UV simultaneously. In this case, the use of the UV group reduces the unnecessary time to search for the location of an emergency or disaster. For these reasons, the development of multi-position methods of operational monitoring carried out with UV is currently an urgent and appropriate task.

In this regard, the task is to develop a technique for combining multi-angle images formed by the onboard equipment of a multi-position system of optical locators. A distinctive feature of the proposed technique is the combination of the method of high-speed pixel clustering with the method of combining different-angle images along the contour.

To solve the problem of image aggregation and highlight useful information on them, the methods of image segmentation, image pixel clustering [1, 2], correlation-extreme methods of combining images [3], methods of multi-position information aggregation using data fusion methods [4], methods of specialized representation and compression of high-resolution images [5, 6] are considered.

II. Multi-Position System Of Unmanned Vehicles Used For Information Processing And Aggregation

The UV group is used to solve the tasks of rapid search of emergency zones, monitoring of hard-to-reach territories and objects of interest in various areas [8, 9]. Their main criterion is the efficiency of execution of the task, which imposes a requirement for monitoring.

One of these requirements is the speed of algorithms, which sometimes entails the use of high-speed computing resources and the development of fast algorithms for processing the information received from the elements of a multi-position system. In addition, in such systems, it is necessary to provide a reliable noise-resistant wireless channel for information exchange between several elements of a multi-position UV system in a complicated electromagnetic environment [10].

To implement such distributed systems (Fig. 1) of data collection and complex processing requires it is necessary first of all to determine the type of information collected from location devices on board the UV.

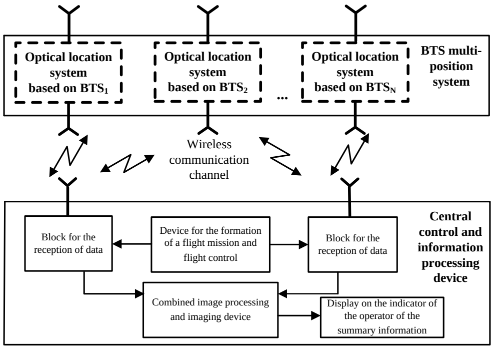


Fig. 1. Scheme of a multi-position UV system

The information collected is usually high-resolution images obtained from optical location systems, radar portraits [11], radar images generated by synthesizing the antenna aperture [12], video frame stream, etc. Thus, the essence of the functioning of the multi-position UV system is as follows. The UV processing and control center receives multi-angle images of the observed zones from several sides via a wireless communication channel from each optical-location station. Further, the processing center combines images to perform various tasks, such as mapping large areas, selecting, locating, recognizing and classifying objects, etc. [13].

In such multi-position systems of unmanned vehicles, the task of integrating information is not trivial enough. Next, it is proposed to consider the developed technique, which consists in applying the method of high-speed clustering of pixels when combining multi-angle images of hardware-generated elements of a multi-position UV system.

III. Model Of High-Speed Clustering Of Image Pixels

The classic Ward`s method [14] takes special place among the cluster analysis methods applicable in digital image processing. It handles both grayscale and color images. The method uses the error sum of squares (ESS) to evaluate the quality of clustering. The lower the ESS value, the better the quality of splitting image pixels into separate clusters (colors). The Ward's method returns adequate results, both in terms of subjective visual perception, and objectively numerically in terms of the ESS value.

However, the Ward's method is characterized by a disadvantage in the form of computational complexity, which prevents the direct application of this method in the processing of digital images. The computational complexity of this method increases at least quadratically with the number of clusters considered. Therefore, in [1, 2], a modification of the classical Ward's method is proposed, which allows to overcome the problem of computational complexity by dividing the entire image processing process into three consecutive stages.

Figure 2 shows a three-stage flowchart of a quasi-optimal clustering algorithm for building a hierarchy of clusters (segments). For a segment, in contrast to a cluster, the adjacency rule for the combined sets of pixels is strictly kept. In the first stage "a", a rough segment hierarchy is quickly constructed. It can be generated in two ways.

The first option is to use the Mumford-Shah model [15, 16]. This method combines a pair of adjacent sets of pixels at each step. In the first step of building the segment hierarchy, each pixel is a segment. At each subsequent step, segments are enlarged by combining a pair of adjacent segments. At the final step, all pixels are combined into a single segment.

The second option is to use the classical Ward's method for image parts [17]. The original image is divided into "cells" by a regular grid. Each "cell" is considered as an independent image. Within each such independent image, pixels are clustered using the classic Ward's method [14]. In the last steps, the hierarchies of pixel clusters of the corresponding "cells" are combined.

The first stage "a" (see Fig. 2) passes to the second stage "b" the hierarchy of connected segments specified by two tables. The first table stores pointers of vertexes to combine, values of ESS of joins, sums of squares, and average brightness values of segments. The second table shows a sequence of combined pairs of segments.

The second stage "b" of the quasi-optimal clustering algorithm forms N_{SP} superpixels. The N_{SP} parameter specifies a fixed number of colors (clusters) to perform the quality improvement procedure for. The range of values for the N_{SP} parameter is from 1 to N, where N is the total number of pixels in the image. A larger value of the N_{SP} parameter corresponds to a larger number of colors (clusters) for which the hierarchical structure is reformatted. Accordingly, a more detailed improvement of the quality of the specified image split is performed for a larger N_{SP} value. Two algorithms have been developed for this stage: the *SI*-method (Segmentation Improvement) [2] and the *K-meanless* method (K-means-without-means method) [18]. The *SI*-method consists in dividing one segment into its two components and combining the other two segments into one. The *K-meanless* method extracts some pixels from one set and assigns them to another. Both methods use the ESS value to evaluate the quality of the resulting split. The iterative process of intermediate quality improvement ends when the segments, whose restructuring results in a drop in the ESS value, comes to end what correspond to the split quality improvement end.

The second stage "b" has many software implementations. Their number is determined both by the possibility of combining a pair of basic *SI* and *K-meanless* methods (separately, sequentially, cyclically), and by the versions of the methods themselves (segmental, cluster).

The third stage "b" clusters previously formed superpixels using the classical Ward's method.

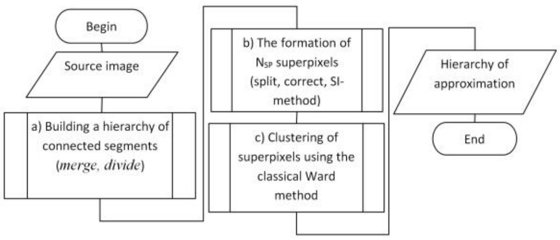


Fig. 2. A three stage scheme for overcoming the computational complexity of the classical ward method

Thus, in order to implement the possibility of the classical Ward's method, it is necessary to perform following preprocessing: prepare the image data structure (stage "a") and improve the quality of this structure in the interim (stage "b"). These preprocedures overcome the disadvantage of the Ward's method associated with computational complexity.

A. Software and algorithmic tools of the model

Four operations on sets of pixels (clusters, segments) form the basis of the software and algorithmic tools of the high-speed pixel clustering scheme [1, 2]: the "merge" operation, the "divide" operation, the "split" operation, and the "correct" operation. All operations minimize the ESS E or the standard deviation σ related by formula:

$$E=3N\sigma^2,$$

where N is the total number of pixels in the image, and 3 is a coefficient indicating the number of color components.

The "merge" operation combines clusters 1 and 2 (adjacent segments 1 and 2) with the number of pixels n_1 and n_2:

$$\Delta E_{merge} = \frac{n_1 n_2}{n_1 + n_2} \|I_1 - I_2\|^2,$$

where I_1 and I_2 are the three-component average pixel brightness values of clusters (segments) 1 and 2, $\| \ \|^2$ – square of the Euclidean difference.

The criterion for selecting a pair of sets 1 and 2 to combine from the set of available ones is the concomitant minimum increment of ESS:

$$(1,2) = \arg\min \left\{\Delta E_{merge}(1,2)\right\}.$$

The *"divide"* operation divides the set of pixels 1 into its two component sets 1' and 1":

$$\Delta E_{divide}(1) \equiv -\Delta E_{merge}(1', 1'') .$$

The *"split"* operation selects a subset of k pixels from the set 1 (with the number of pixels n_1) ($k < n_1$):

$$\Delta E_{split} = -\frac{kn_1}{n_1 - k}\|I - I_1\|^2, \ \Delta E_{split} \le 0,$$

where I and I_1 are the three-component average brightness of *n1* and k pixels.

The *"correct"* operation classifies k pixels by excluding them from the set 1 (with the number of pixels n_1) and then assigning them to the set 2 (with the number of pixels n_2):

$$\Delta E_{correct} = \frac{kn_2}{n_2 + k}\|I - I_2\|^2 - \frac{kn_1}{n_1 - k}\|I - I_1\|^2,$$

where I is the average value of the reclassified k pixels, I_1, I_2 is the average value of pixels of clusters 1 and 2.

The criterion of the *"correct"* operation execution is the maximum reduction of the ESS increment value:

$$(1,2,k) = \arg\min \Delta E_{correct}(1,2,k), \ \Delta E_{correct} \le 0.$$

A pair of *"merge&divide"* operations forms an *SI*-method for improving the quality of a given partition. The method divides segment 1 into its two components and combines two other 2 and 3 that do not match the 1 into one. The pixel set adjacency rule is critical for a segment. The *SI*-method is performed iteratively by the criterion of the maximum drop in the value of the increment of the ESS [2]:

$$\Delta E(1,2,3)= \Delta E_{divide}(1) + \Delta E_{merge}(2,3);$$
$$(1,2,3)=\mathrm{argmin}(\Delta E(1,2,3));\ \Delta E(1,2,3)<0.$$

The iterative process of *"merge-and-divide"* is repeated as long as there is such a triple of segments 1, 2, 3, the execution of the operation on which leads to a drop in the total value of the ESS in the specified partition. During the execution of the *SI*-method, the total number of segments remains constant.

The *"correct"* operation forms the *K-meanless* method [17].

The *"merge"* and *"divide"* operations are used to build a hierarchy. The *"split"* and "*correct*" operations transform a hierarchy that has already been built. The hierarchy of pixel clusters is considered to be set if for each cluster consisting of at least one pixel, a pair of clusters is set into which the given cluster is divided.

B. Results of image clustering and segmentation

Figure 3 shows the final part of an example of a sequence of piecewise-constant partitions of the source image. The partitions are obtained by clustering pixels as in the three-step scheme in figure 2. In this case, the value of the auxiliary parameter N_{SP} of the number of superpixels, which sets the accuracy of calculations, is 1000. The accuracy parameter can take values from 1 to N. For N_{SP} = 1, the scheme (Fig. 2) operates in pure segmentation mode, as in the Mumford-Shah model [17]. If N_{SP} = 1000, in the regime of quasi-optimal clustering. When N_{SP} = N – in pure pixel clustering mode, which is available for images of small sizes, due to the high computational complexity of the original Ward's method (for 512x512 images, the calculation time exceeds 20,000 seconds).

As the source image for processing, the left image is taken from a pair of aerial images of the landscape of the underlying surface, intended for further aggregation into a single image. Figure 3 shows the scaled source image in the upper-left corner. For this source image of 1728x1350 pixels, only 2332800 partitions are available. To the right of it - part of a sequence of clustered partitions, according to the diagram in figure 2. There are 20,6,5,4,3,2 and 1 partitions into clusters. Above each such partition, the number of colors (clusters) that it consists of is indicated. Under the partition, the ESS value corresponding to this partition is specified.

For a separate left, right (Fig. 5) and composite image of the source images, a series of partitions from 1 to 20 colors were obtained in two different modes: in the mode of pure segmentation and in the mode of quasi-optimal pixel clustering. In figure 4, solid lines represent sequences of segmented partitions (N_{SP} = 1). Dotted lines represent the results of clustering pixels in the proposed three-stage scheme (N_{SP} = 1000). The color (green, red, and blue) indicates different images that were processed in two ways. The lines corresponding to the results of processing docked images into a single image are marked in green. The lines corresponding to the results of processing a single left image are marked in red; blue color – separate right image. The number of colors (number of clusters) in the partitions is set on the abscise axis of the graph in Fig. 4. On the ordinate axis, the value of the ESS.

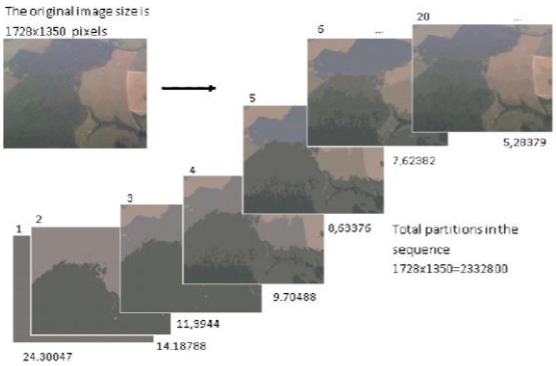


Fig. 3. Example of a sequence of piecewise constant partitions for a given image

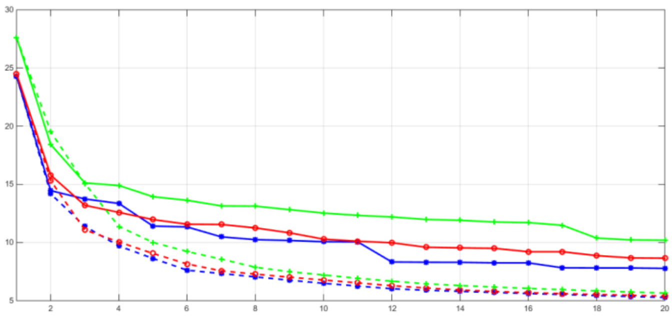


Fig. 4. Graphs of the dependence of the COE values of different types of partitions on the number of colors (clusters or segments) in the partition

According to the graph in Fig. 4, the proposed three-stage scheme gives the best results in terms of the ESS value. The error is less than for pure segmentation. In this regard, it is advisable to use the scheme of quasi-optimal high-speed clustering of image pixels for the subsequent procedure of combining color-reduced multi-angle images taken by onboard optical-location systems.

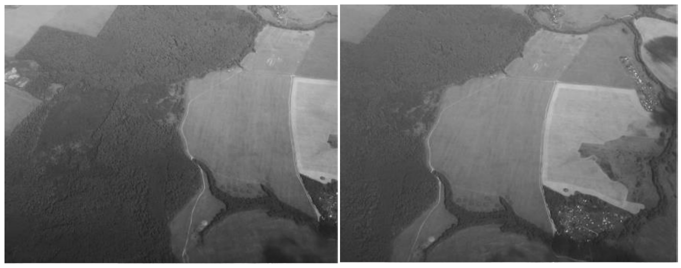
IV. Experiments On Combining Images

This section shows the feasibility of application of the algorithm of quasi-optimal pixel clustering [1, 2], described in section 2, for more accurate aggregation of images captured by the equipment of optical-location stations of the multi-position UV system.

The spatial transformation procedure is used to get a combined image. Spatial transformation procedure is used when combining images of a single scene obtained from different sources of location information or sequentially formed by a single source, such as a high-resolution video camera. In this case, the problem of combining images is posed as the problem of finding some functional transformation that achieves the greatest match based on correlation-extreme methods. Next, the developed technique of information aggregation, which allows combining different-angle location images into one is going to be illustrated. This paper considers two ways to do this. The first is a way to combine images based on the found reference points. The second is a method being developed for combining the selected contour areas by points using the method of quasi-optimal pixel clustering.

A. Reference point method

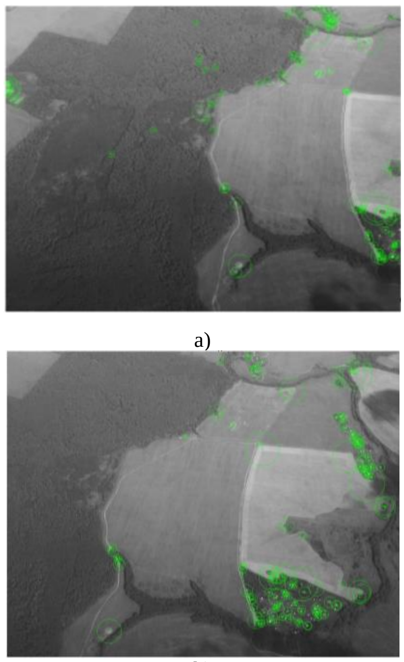
Figures 5a and 5b show two images of different sizes obtained as a result of experimental flights of two UVs. These images are formed from different angles by the optical-location system equipment. Further, these images (frames) (see Fig. 5) are transmitted via a wireless high-speed noise-proof channel [5] to the data processing and aggregation center from UV_1 (left image) and UV_2 (right image). For convenience of further processing and speed of calculation, these images are presented in grayscale.



a) b)

Fig. 5. Images obtained by UVs equipment

The procedure for combining images into a single image using the reference point method based on searches and selects characteristic points in the images (Fig. 6a and 6b). Based on the characteristic points found in both images, they are compared (Fig. 7), and then they are combined (Fig. 8) [18, 19].



a)

b)

Fig.6. Displaying reference points on multi-angle images

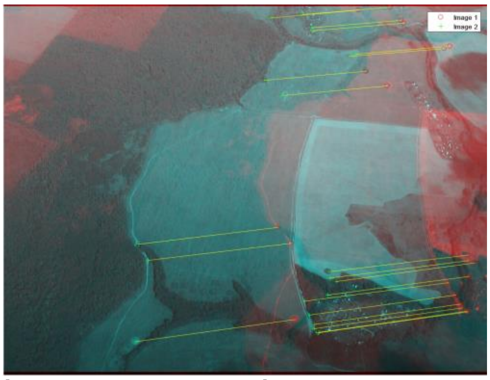


Fig.7. Overlaying two images into one and point mapping

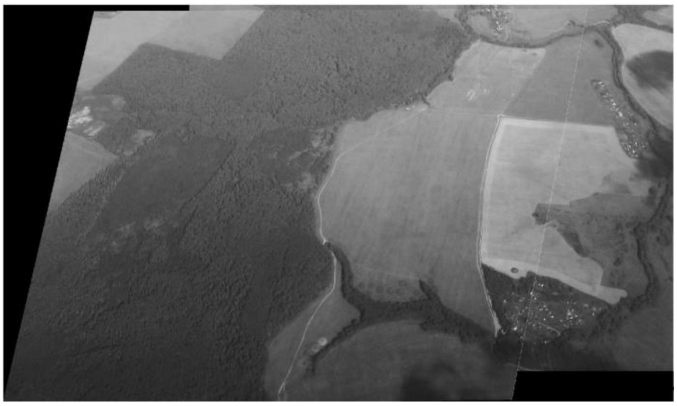


Fig. 8. Combined image

Figure 8 shows a combined image suitable for further processing. The considered method of reference points has a number of limitations. In particular, it is not applicable if the found number of reference points is less than four. On the other hand, the method is easy to use and implement. It is also applicable for both color and grayscale images.

The disadvantages of the method include the fact that the method has great computational complexity. For example, image processing of size 1440x900 (using the developed implementation of the algorithm) cannot be performed in real time on a regular personal computer. Some reference points can be deleted as a result of preliminary filtering of the image. This may affect the further solution of the image matching problem. Also, the method does not work correctly in cases if the lighting conditions are radically different (for example, day and night), and if the image has a fractal or similar structure (for example, a brick wall).

Due to these disadvantages, the second method of combining, which is not characterized by the limitations of the first, is considered.

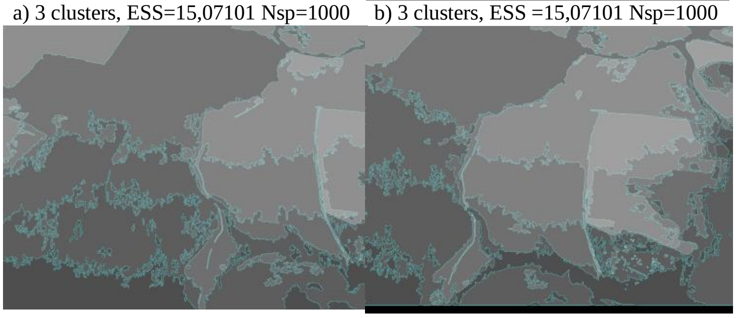
B. Method for matching contour points

The second method is based on combining images on contour points. The essence of the method is as follows. The method of combining images is to form a value that measures the correlation between two matrix structures (clustered images in grayscale). For these structures, there is some functional transformation, at the application of which the value of the correlation function will reach its maximum when used.

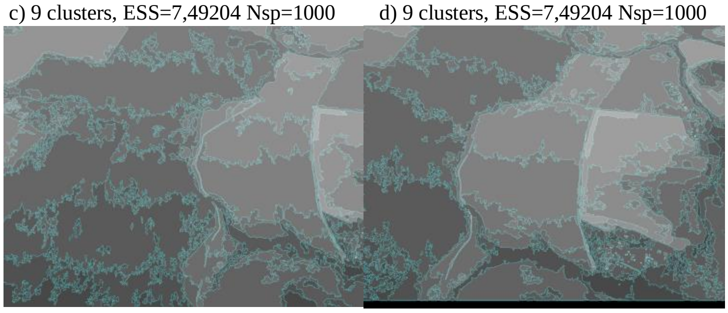
A distinctive feature of the considered method of matching contour points from the method of reference points is its iteration. At each iteration, the position of the found contour points is clarified, which in turn allows you to more fully determine the type of functional transformation. The combining process consists of the following steps.

In the first step, both images are converted to a clustered view, using the method described in section 2. Then the internal contours of the clustered images are highlighted.

a) 3 clusters, ESS=15,07101 Nsp=1000 b) 3 clusters, ESS =15,07101 Nsp=1000



c) 9 clusters, ESS=7,49204 Nsp=1000 d) 9 clusters, ESS=7,49204 Nsp=1000



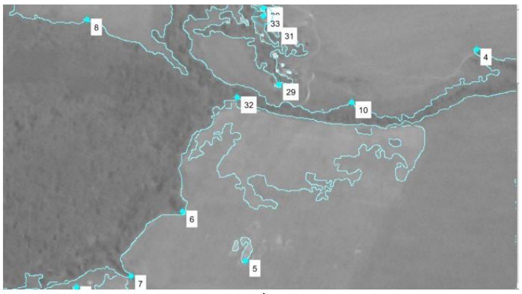
e) 18 clusters, ESS=5,84563 Nsp=1000 f) 18 clusters, ESS=5,84563 Nsp=1000

Fig. 9. Clustered images with selected contours

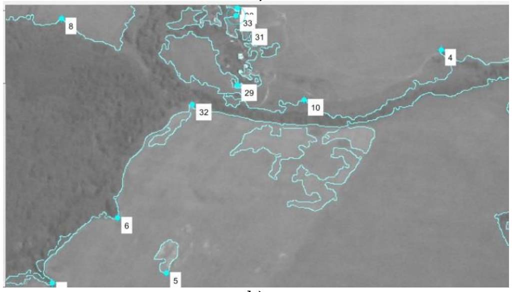
Figure 9a-9f shows pairs of clustered images with selected polygons. However, each pair has a different level of detail (the number of colors or clusters). The procedure of processing them is carried out as follows. The left and right images were docked into one. Next, the quasi-optimal clustering described in section 2 was applied to this docked image. Then the processed docked image was again divided into left and right. And then the contours of the characteristic areas were highlighted on the left and right images. This method of processing allows you to select areas of pixels that are similar in structure in the same way on multi-angle images

Figure 10 shows parts of the total observed area of two multi-angle images (a-left, b-right) at an enlarged scale. The enlarged fragment shows the outlines of various areas.

In the second step, the characteristic points of similar areas were defined on these contours. The number of defined points is about 30.



a)

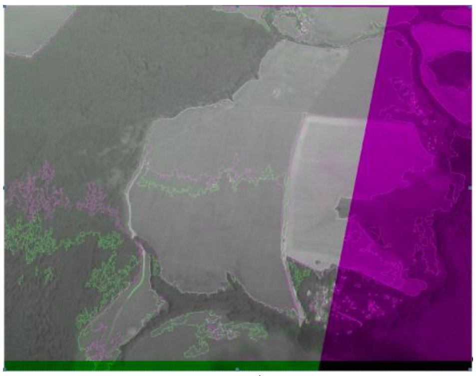


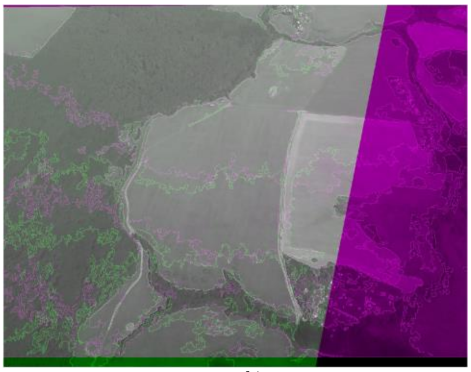
b)

Fig. 10. Definition of contour reference points

Then, in the third step, some functional transformation is selected to these points of the contour, in which the value of the correlation function takes the maximum value. This functional transformation is the initial one for subsequent iterations. At the current initial clustering parameters (Fig. 9a, 9b) and with the found functional transformation, a combined image was obtained (Fig. 11a). The transition to subsequent iterations is advisable in the case of unsatisfactory subjective assessment of the quality of combining the generated image.

Similar actions (steps 1-3) are performed for partitions with a large number of clusters (with greater detail). In this case, the search range for both the contour reference points themselves and the functional transformation will be narrowed and refined. At the same time, the number of reference points on the contour increases. The accuracy of determining their position also increases. Due to these reasons, the functional transformation is refined by the correlation method. After the position of the reference points is clarified and a refined functional transformation of the original multi-angle images is found, the operation of combining them is performed (Fig. 11b). Then again, a subjective assessment of the quality of the combined image is made.


a)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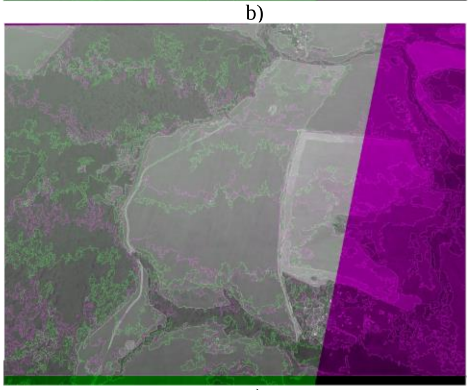
c)

Fig. 11. The combined image

Thus, the technique of combining images based on contour points consists in preliminary reducing the number of colors of the source images using a three-stage method of quasi-optimal clustering. Then you search for the reference points of the selected contours and determine the functional transformation with an assessment of the degree of correlation of the combined images. Then the process is repeated for clustered images with more detail. Both the position of the contour reference points and the desired functional transformation itself are specified until a subjective assessment of the quality of the alignment is acceptable.

V. CONCLUSION

In present paper, the algorithm for quasi-optimal pixel clustering was proposed to form starting sets of partitions for the subsequent procedure of combining multi-angle images. The experiments on color reduction of multi-angle images in pure segmentation and quasi-optimal clustering modes were carried out. It has been established that the results of quasi-optimal clustering of image pixels are acceptable both subjectively in visual perception and objectively in terms of the error sum of squares.

The paper proposes the way for processing multi-angle images which consists in tie-in a series of images into a sole one image with subsequent clustering of this single image. This processing way selects similar areas of pixels on different images in the same manner. The experimental results confirmed the possibility of combining clustered images which were obtained from different angles.

The technique for combining images including algorithm for pixel clustering and a method for selecting matching contour points, have been developed. This technique is applicable for airborne location stations of multi-position system of the UV for the purpose of combining the information.

In present paper, two ways of combining images are considered: the way of reference points and the way of contour points. It is important to note that the second combing way increases the number of contour points with the rise of cluster number in partition in contrast to the first combining way. At each subsequent iteration, detalisation of image partition rises, followed by increase of the number of contour points and refinement of the desired functional transformation. It should also be noted that refining algorithm of the functional transformation should be repeated with increase image partition detalisation until the number of contour points becomes stable, or until the degree of assessed quality of the combined image is acceptable. Multiple experiments show the feasibility of selecting contour points in clustered images.

The research results can be adapted for testing the algorithms of clustering and combining of the images generated from video stream during field tests. The obtained in present paper results are significant primarily for the organizations performing operational search and rescue operations in emergency and natural and man-made disasters, as well as for environmental monitoring of hard-to-reach places. The results of optical image combining presented in this article are the basis for further research on the aggregation of information from radar systems and optical location systems, which is appropriate for modern operational monitoring systems.

VI. Acknowledgments

The reported study was funded by a grant of Russian Science Foundation (project № 19-79-00303).

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