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Nenashev, Vadim, Shepeta, Dmitry

Vadim A. Nenashev, Dmitry A. Shepeta, "Mathematical models and algorithms for modeling the location signals reflected from the underlying surfaces of the earth, sea, and coastal waters," Proc. SPIE 11150, Remote Sensing of the Ocean, Sea Ice, Coastal Waters, and Large Water Regions 2019, 111501V (14 October 2019); doi: 10.1117/12.2547871

Event: SPIE Remote Sensing, 2019, Strasbourg, France

Mathematical models and algorithms for modeling the location signals reflected from the underlying surfaces of the earth, sea and coastal waters

Vadim A. Nenashev[a], Dmitry A. Shepeta[a]

[a]Saint-Petersburg State University of Aerospace Instrumentation

67, Bolshaya Morskaia str., Saint-Petersburg, 190000, RUSSIA

ABSTRACT

In developing the on-board equipment of aircraft, used radar maps for navigation, there is a need for mathematical model's signals, reflected from the Earth's surface, sea surface and coastal edge. Traditionally, using a theoretical construct, stochastic signals were used as such models and its fluctuations were described by Rayleigh and Rayleigh-Rice. These models are used both for simulation signals, reflected from the Earth's surface, and for signals, reflected from the surface of the sea. At low resolution capability of on-board radars, the similar models describe quite well the statistical characteristics of fluctuating signals. Modern on-board locator has high resolution capability and the Rayleigh and Rice's models can no longer be used in the synthesis and simulation of modern on-board navigation systems.

In this paper, we propose an approach to the construction of models of radar signals using both theoretical constructs and experimental data that allows you to take into account the features of reflection of radar signals from small plots of the Underlying Surface of earth and sea. Along with it the correlations between the individual sections and anisotropic reflections are taken into account when observing sites with different angles. The reflections from the sea surface approximated by a log-normal law, reflections from the earth's surface at the sight of the manifold types of surface by the Beckmann and Weibull laws, special cases of which are the laws of Rice, Rayleigh and Hoyt. As the reflection model of the edge uses the distribution law of the vector sum of the signals, reflected from the elementary areas of earth and sea, getting in the resolution cell. In this case, the law of distribution of the total vector is subject to a law similar to Huber's law, in which the above-mentioned laws of distribution of reflections from the earth and the sea are used as the basic distributions. Previously used reflection models turned out as special cases of the proposed models.

These models and the modeling algorithms developed for them can be used in the development and research of high-precision methods of radar monitoring for the purposes of environmental reconnaissance, forecasting and prompt prevention of natural and man-made emergency situations. In addition, for testing the operating modes of the equipment of unmanned aerial vehicles, including for multi-position radar systems.

Algorithms for modeling location signals based on mathematical models using experimental data of reflections from various types of underlying land and sea surfaces, as well as coastal edges (coastal waters), allow us to bring the results of computer experiments to the results of actual tests of radio-electronic equipment. This reduces the time and reduces the cost of design by reducing the semi-natural and full-scale tests.

Keywords: random field, coastal waters, remote sensing, spatial-correlation properties, simulation model of echo-signal, high resolution, SAR images, reflected from the underlying surfaces.

1. INTRODUCTION

At present, the characteristics of multichannel complexes containing complex nonlinear devices are found by methods of mathematical modeling on computing devices. In this case, it is necessary to build models of all input signals of devices, as well as realize the algorithm of data processing. If simulation models are built on the basis of experimental data, then the method of mathematical modeling is a computer experiment. This allows «experimentally» to receive the characteristics of the projected systems for situations that are closest to the real operating conditions of the created equipment.

Remote Sensing of the Ocean, Sea Ice, Coastal Waters, and Large Water Regions 2019,
Charles R. Bostater, Xavier Neyt, Françoise Viallefont-Robinet, Eds., Proc. of SPIE Vol. 11150,
111501V · © 2019 SPIE · CCC code: 0277-786X/19/$21 · doi: 10.1117/12.2547871

When developing mathematical models for the reflection of radar signals from underlying surfaces of the earth [1], sea [2] and shoreline [3], it is necessary to take into account both the non-Gaussian nature of fluctuations of the reflected signals and the spatiotemporal correlation between the reflected signals [4]. Because of this, it is convenient to use the reflection model in the form of a stochastic field, which, in turn, can be represented as a matrix, whose elements are signals reflected from individual resolution elements of the onboard location system, as a mathematical divide of the underlying surface. In this case, with an increase in the resolving power of locators, spatial correlation dependencies between matrix elements increase, and the process of modeling reflections actually comes down to modeling a sequence of stochastic matrices connected by time correlation dependencies.

In this paper, we present a mathematical model for modeling random vectors and matrices with given statistical characteristics. The presented modeling algorithm is oriented to statistical problems, their synthesis was carried out taking into account the subsequent implementation on a computer, which determined their high efficiency in comparison with the known algorithms.

2. FEATURES OF SYNTHESIS OF NONLINEAR MULTICHANNEL DISCRETE SHAPING FILTERS

The methods of synthesis of nonlinear multichannel discrete forming filters (NMDFF) considered in [5], which are used for modeling random vectors and matrices with given statistical characteristics, allow a natural generalization to the modeling of anisotropic Gaussian and non-Gaussian random fields [6].

The only restriction for the direct use of the synthesis method outlined in this article is the restriction on the factorization of the correlation functions with respect to the spatial coordinates for the generating non-Gaussian field of the corresponding Gaussian field (in the general case, anisotropic).

However, such a limitation is not significant from a practical point of view, since the results of experimental field research are often represented by sections of their Space-correlation function - for each of the coordinates and in time. Therefore, the imposed restriction always allows us to construct a statistical field model that does not contradict the experimental data.

It should be noted that the assumption of the factorization of the space-time correlation function of the field refers only to the generating Gaussian field. A simulated non-Gaussian field can have a spatiotemporal correlation function, which does not factor. Moreover, the generating Gaussian field in practical applications can often be decomposed into orthogonal components, which, due to the normality of the field, are statistically independent. In this case, the factorization is natural and, therefore, does not narrow the problem of modeling the generating field.

In the synthesis NMDGF in [6], the assumption was made that the normalized temporal statistical characteristics of the processes in channel shaping filters are identical. This assumption dramatically improves the speed of simulation algorithms, and it is not too restrictive for the practical use of synthesized algorithms since such assumptions are carried out in many practical cases (and their implementation is often simply implied and even not specified). Therefore, in the synthesis of field simulation algorithms, we assume that this assumption is fulfilled. We note that the mean values and variances of the processes can vary, which is used in practical situations. Such an assumption (on the uniformity of the normalized statistical characteristics for each of the coordinates of the field) will also be made with respect to spatial coordinates.

We confine ourselves to the explicit recording of simulation algorithms for one-dimensional (vector) and two-dimensional (matrix) fields. Since these fields are used in the form of mathematical models of observable physical processes. It is these fields that can be represented by echoes from the shoreline and underlying surfaces (land and sea).

3. ALGORITHM FOR MODELING NON-GAUSSIAN ANISOTROPIC FIELDS

For the algorithms for modeling non-Gaussian fields, we introduce the following notation: V are the elements of the non-Gaussian field; U - elements of the generating Gaussian field; (X, Y) - orthogonal coordinates of the field U; $r^{(X)}$, $r^{(Y)}$ - the corresponding normalized correlation functions (elements of the correlation matrices) of the coordinates of the generating Gaussian field; $f(.)$ - functional transformation corresponding to the required distribution density of the simulated field; ξ - jointly independent random normal (pseudo-random) variables with zero mean and unit variance. All the quantities entered can have an arbitrary number of indices in the general case [1, 2].

In our particular case, the constraints are no more than two-dimensional, time-varying fields, the number of indices does not exceed three, and one of the indices is necessarily temporary.
The algorithm for modeling a one-dimensional field (vector) in these notations can be represented in the form [1, 2, 6]

$$\begin{cases} X_{i,t} = -\sum_{l=1}^{i-1} (D_{l,i}^{(X)} / D_{l,l}^{(X)}) \cdot X_{l,t} + \sqrt{D_i^{(X)} / D_{i-1}^{(X)}} \cdot \xi_{i,t} \,, \\ U_{i,t} = \sum_{s=1}^{N} a_s \cdot U_{i,t-s} + \sum_{d=0}^{N-1} b_d \cdot X_{i,t-d} \,, \qquad\qquad i = 1,2,\ldots M^{(X)}, \\ \tilde{V}_{i,t} = f(\overline{U}_{i,t} + \sigma_{i,t} \cdot U_{i,t}), \end{cases} \tag{1}$$

where $i=1,2,\ldots,M^{(X)}$, $X_{1,t} = \xi_{1,t}$ during, $\forall t$, $t = \ldots -3,-2,-1,0,1,2,3,4,\ldots$, $D_1^{(X)} = D_0^{(X)} = 1$, $D_{l,l}^{(X)} = D_{l-1}^{(X)}$, $l = 1,2,\ldots M^{(X)}$

$$D_l^{(X)} = \begin{vmatrix} 1 & r_1^{(X)} & r_2^{(X)} & \cdots & r_{l-1}^{(X)} \\ r_1^{(X)} & 1 & r_1^{(X)} & \cdots & r_{l-2}^{(X)} \\ r_2^{(X)} & r_1^{(X)} & 1 & \cdots & r_{l-3}^{(X)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{l-1}^{(X)} & r_{l-2}^{(X)} & r_{l-3}^{(X)} & \cdots & 1 \end{vmatrix}, \tag{2}$$

The density of the distribution $\tilde{V}_{i,t}$ is determined by the functional transformation $f(.)$.
The algorithm (2) can be conveniently interpreted as follows:
1) The upper expression in (1) transforms for each fixed t the vector $\boldsymbol{\xi}_{M^{(X)},t} = (\xi_{1,t}, \xi_{2,t}, \ldots \xi_{M^{(X)},t})$ - vector of jointly independent normally distributed random variables with zero means and unit variances into a vector $\mathbf{X}_{M^{(X)},t} = (X_{1,t}, X_{2,t}, \ldots X_{M^{(X)},t})$ of normally distributed random variables, also with zero means and unit variances, but with a normalized correlation matrix $\mathbf{D}_{M^{(X)}}^{(X)} = \left\| D_{M^{(X)}}^{(X)} \right\|$, the matrix $\mathbf{D}_{M^{(X)}}^{(X)}$ does not depend on t. This transformation is conditionally written as [6]

$$\boldsymbol{\xi}_{M^{(X)},t} \overset{\mathbf{D}_{M^{(X)}}^{(X)}}{\Rightarrow} \mathbf{X}_{M^{(X)},t} \,. \tag{3}$$

2) The average expression in (1) converts each of the elements of the vector $\mathbf{X}_{M^{(X)},t} = (X_{1,t}, X_{2,t}, \ldots X_{M^{(X)},t})$ into a sequence $\mathbf{U}_{i,t} = (\ldots -2,-1,0,1,2,3,\ldots)$ by means of a linear discrete shaping filter with [5] coefficients ($\mathbf{a}_N$, $\mathbf{b}_N$)=(a_1, a_2, …, a_N; b_0, b_1, …, b_{N-1}) that determine the normalized time. The correlation function $r^{(B)}(t_1, t_2) = r^{(B)}(|t_1 - t_2|) = r^{(B)}(\tau)$ of the sequence for each of the elements of the vector $\mathbf{X}_{M^{(X)},t}$; here, by virtue of the assumptions made, $r^{(B)}(\tau)$ does not depend on the first index of the element of the vector $\mathbf{X}_{M^{(X)},t}$. Thus, this transformation forms a vector normal process, or, in other words, forms $M^{(X)}$ normal correlated processes, the time correlation function of each of which is equal

$r^{(B)}(\tau)$, and the mutual correlation function of processes $X_{i,t_1} = X_i(t_1)$ and $X_{j,t_2} = X_j(t_2)$ is equal to $r^{(B)}(|t_1 - t_2|) \cdot r^{(X)}(|i-j|)$, $i, j = 1,2,...M^{(X)}$. We denote this transformation as [6].

$$X_{M^{(X)},t} \overset{(\mathbf{a}_N, \mathbf{b}_N)}{\Longrightarrow} \mathbf{U}_{M^{(X)},t}, \tag{4}$$

3) The expression in the third line (1) defines a nonlinear functional transformation $M^{(X)}$ of normal stationary and stationary connected processes in non-Gaussian non-stationary $M^{(X)}$ processes $V_{i,t_1} = V_i(t_1)$, or in other words, a non-Gaussian vector process $\tilde{\mathbf{V}}_{M^{(X)},t}$. The distribution densities, correlation and mutual correlation functions of the process $\tilde{\mathbf{V}}_{M^{(X)},t}$ are calculated from the corresponding expressions given in [5]. We denote this nonlinear conversion, including parameters $(\overline{\mathbf{U}}_{M(X),t}, \boldsymbol{\sigma}_{M(X),t}) = (\overline{U}_{1,t}, \overline{U}_{2,t}, ...\overline{U}_{M(X),t}; \sigma_{1,t}, \sigma_{2,t}, ...\sigma_{M(X),t})$ such as

$$\mathbf{U}_{M^{(X)},t} \overset{f(.,\overline{\mathbf{U}}_{M^{(X)},i}, \boldsymbol{\sigma}_{M^{(X)},i})}{\Rightarrow} \tilde{\mathbf{V}}_{M^{(X)},t} \tag{5}$$

In this notation, the algorithm (1) can be rewritten in the form [6]

$$\boldsymbol{\xi}_{M^{(X)},t} \overset{\mathbf{D}^{(X)}_{M^{(X)}}}{\Rightarrow} \mathbf{X}_{M^{(X)},t} \overset{(\mathbf{a}_N, \mathbf{b}_N)}{\Rightarrow} \mathbf{U}_{M^{(X)},t} \overset{f(.,\overline{\mathbf{U}}_{M^{(X)},i}, \boldsymbol{\sigma}_{M^{(X)},i})}{\Rightarrow} \tilde{\mathbf{V}}_{M^{(X)},t} \tag{6}$$

Expression (6) allows, by adding new indices, to write purely formal algorithms for modeling fields of higher dimension. In the paper, for the reasons indicated above, we confine ourselves to recording (formal - in the form of (6) and expanded - in the form of (1) algorithms for modeling one-dimensional and two-dimensional fields, vectors, shoreline modeling, and matrices for modeling reflections from the underlying surfaces of the earth and the sea, respectively.
The algorithm for modeling a two-dimensional field (matrix) is formally obtained from expression (6), by introducing a coordinate Y, an additional transformation $\mathbf{X}$ in $\mathbf{Y}$ and corresponding indexing of notation [6]

$$\boldsymbol{\xi}_{M^{(X)},M^{(Y)},t} \overset{\mathbf{D}^{(X)}_{M^{(X)}}}{\Rightarrow} \mathbf{X}_{M^{(X)},M^{(Y)},t} \overset{\mathbf{D}^{(Y)}_{M^{(Y)}}}{\Rightarrow} \mathbf{Y}_{M^{(X)},M^{(Y)},t} \overset{(\mathbf{a}_N, \mathbf{b}_N)}{\Rightarrow} \mathbf{U}_{M^{(X)},M^{(Y)},t} \overset{f(.,\overline{\mathbf{U}}_{M^{(X)},M^{Y},i}, \boldsymbol{\sigma}_{M^{(X)},M^{Y},i})}{\Rightarrow} \tilde{\mathbf{V}}_{M^{(X)},M^{(Y)},t} \tag{7}$$

When expanded, (7) it looks like [1, 2]

$$\begin{cases} X_{i,j,t} = -\sum_{l=1}^{i-1} (D^{(X)}_{l,i} / D^{(X)}_{l,l}) \cdot X_{l,j,t} + \sqrt{D^{(X)}_{i} / D^{(X)}_{i-1}} \cdot \xi_{i,j,t}, \quad i = 1,2,...M^{(X)}, \\ Y_{i,j,t} = -\sum_{p=1}^{j-1} (D^{(Y)}_{p,j} / D^{(Y)}_{p,p}) \cdot Y_{i,p,t} + \sqrt{D^{(Y)}_{j} / D^{(Y)}_{j-1}} \cdot X_{i,j,t}, \quad j = 1,2,...M^{(Y)}, \\ U_{i,j,t} = \sum_{s=1}^{N} a_s \cdot U_{i,j,t-s} + \sum_{d=0}^{N-1} b_d \cdot Y_{i,j,t-d}, \\ \tilde{V}_{i,j,t} = f(\overline{U}_{i,j,t} + \sigma_{i,j,t} \cdot U_{i,j,t}), \end{cases} \tag{8}$$

where, analogically (1), $i=1,2,\ldots,M^{(X)}$, $j=1,2,\ldots,M^{(Y)}$, $X_{1,1,t}=\xi_{1,1,t}$ during $\forall t$, $t=\ldots-3,-2,-1,0,1,2,3,4,\ldots$ and for $\forall t$ $D_1^{(X)}=D_0^{(X)}=D_1^{(Y)}=D_0^{(Y)}=1$, $D_{l,l}^{(X)}=D_{l-1}^{(X)}$, $l=1,2,\ldots M^{(X)}$, $D_{p,p}^{(Y)}=D_{p-1}^{(Y)}$, $p=1,2,\ldots M^{(Y)}$.

Further generalization of modeling algorithms to a large field dimension is evident due to the very convenient representation of the algorithm in the form (7). Such a generalization may be needed when modeling echo signals of atmospheric inhomogeneities, for example, when modeling location signals reflected by hydrometeors, since in this case we are dealing with spatially distributed reflectors.

4. PRACTICAL USE

In the practical use of the above algorithms, one should find a functional transformation $f(.)$ That would reproduce a given distribution law of fluctuations of the reflected signal with a normal distribution of the argument. As the laws of distribution of fluctuations of the envelope of the echo signals of the earth and the sea, different distribution laws are used: log-normal, Weibull, *K*-distribution, composite distributions, etc. [7, 8]. The required nonlinear transformation can be found using the well-known, but somewhat modified, simulation method for reversing the distribution function of the reproduced law, namely, if the distribution function V is $F(x)$, that is, $V \sim F(x)$, then $\eta = F(V) \sim RAV(0,1)$ - has a uniform distribution over the interval (0,1). The random variable $\acute{\eta} = F_{\mathrm{n}}(\xi)$, where ξ is distributed normally $\xi \sim NOR(0,1)$, $NOR(0,1) = F_{\mathrm{n}}(\xi)$ is a normal distribution with zero mean and unit dispersion, also has a uniform distribution on (0,1). Equating $\eta = F(V) = \acute{\eta} = F_{\mathrm{n}}(\xi)$, we find the required functional transformation $V = F^{-1}(\acute{\eta}) = F^{-1}(F_n(\xi)) = f(\xi)$, thus obtaining an algorithm for modeling the given distribution through the standard normal distribution, which is required for the practical use of the algorithms presented in the paper.

When modeling fluctuations in the echo signals of the earth and sea surface in the range gate of the receiving device of the airborne locator, you can use the algorithm for modeling a one-dimensional field represented by expression (1). If the locator is simulated using the telescopic mode when synthesizing the antenna aperture, then it is necessary to use the algorithm for modeling a two-dimensional field represented by expression (8) [9, 10].

The modeling of the echo signal of the land-sea edge is described in detail in [3]. A feature of modeling this portion of the underlying surface is that reflections, both from the earth's surface and from the surface of the sea, get into the resolution element. If fragments of the sea surface with an area of $S_s = \gamma S$ and the earth's surface with an area of $S_e = (1 - \gamma)S$ fall into the resolution element of a locator of area S, then the echo-signal powers of the earth and sea are proportional to these coefficients γ and $(1-\gamma)$. And the reflected signal is formed as the vector sum of the corresponding fragments of the underlying surfaces. Assuming the phase difference between the vectors distributed according to a uniform law, can write an algorithm for modeling the edge echo [3]

$$A=\sqrt{A_{(\mathrm{e})}^2+A_{(\mathrm{s})}^2+2A_{(\mathrm{e})}A_{(\mathrm{s})}\cos(2\pi\eta)}, \tag{9}$$

where $A_{(\mathrm{e})}^2$ and $A_{(\mathrm{s})}^2$ the amplitudes of the echo signal vectors reflected from the land and the sea, respectively, and $\eta \sim RAV(0,1)$ - is evenly distributed over the interval (0,1). In this case, for modeling processes $A_{(\mathrm{e})}^2$ and $A_{(\mathrm{s})}^2$ can use a simulation algorithm (1).

5. CONCLUSION

The research results are synthesized mathematical models and modeling algorithms for non-Gaussian anisotropic fields. These models of random fields are the mathematical equivalents of echo signals, which allow one to take into account the spatial and temporal correlation and spectral characteristics of reflections between resolution elements of various types of underlying surfaces, such as land, sea, and the land-sea edge.

The synthesized algorithms make it possible to take into account the spatial correlation between the echo signals reflected by individual elements of the underlying surfaces, which is especially important when modeling high-precision locators that use the synthesis of the antenna aperture when mapping the terrain and determining the outlines of the coastal edge.

The algorithms do not use any specific laws of the distribution of fluctuations of echo signals, but are universal and can be used with almost any laws, and a method for finding the required nonlinear transformation used to generate sequences of random vectors and matrices with given statistical characteristics is proposed. Regarding the correlation-spectral characteristics of the resulting sequences of vectors and matrices, the use of the nonlinear forming filter method imposes some restrictions on the functional form of these dependencies, namely, to increase the speed of the algorithms, it is necessary to use Markov models of normal random processes, with the help of which the corresponding correlation properties are formed.

The reported study was funded by a grant of Russian Science Foundation (project №19-79-003).

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